

Critical Temperature of Chiral Symmetry Restoration for Quark Matter with a Chiral Chemical Potential

M. Ruggieri^{1,*} and G. X. Peng^{1,2,†}

¹*College of Physics, University of Chinese Academy of Sciences, Yuquanlu 19A, Beijing 100049, China*

²*Theoretical Physics Center for Science Facilities,
Institute of High Energy Physics, Beijing 100049, China*

In this article we study restoration of chiral symmetry at finite temperature for quark matter with a chiral chemical potential, μ_5 , by means of a quark-meson model with vacuum fluctuations included. Vacuum fluctuations give a divergent contribution to the vacuum energy, so the latter has to be renormalized before computing physical quantities. The vacuum term is important for restoration of chiral symmetry at finite temperature and $\mu_5 \neq 0$, therefore we present several plausible renormalization schemes for the ultraviolet divergences at $\mu_5 \neq 0$. Then we compute the critical temperature as a function of μ_5 . The main result of our study is that the choice of a renormalization scheme affects the critical temperature; among the three renormalization schemes we investigate, there exists one in which the critical temperature increases with μ_5 , a result which has not been found before by chiral model studies and which is in qualitative agreement with recent lattice data as well as with studies based on Schwinger-Dyson equations and universality arguments.

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I. INTRODUCTION

Systems with chirality imbalance, namely with a finite chiral density $n_5 = n_R - n_L$ generated by quantum anomalies, have attracted some interest in recent years. In fact gauge field configurations with a finite winding number, Q_W , can change fermions chirality according to the Adler-Bell-Jackiw anomaly [1, 2]. For example in Quantum Chromodynamics (QCD) such nontrivial gauge field configurations with $Q_W \neq 0$ are instantons and sphalerons, the latter being produced copiously at high temperature [3, 4]. In QCD the time evolution of n_5 is governed by the equation

$$\frac{dn_5}{dt} = \frac{g^2 N_f}{16\pi^2} F_a^{\mu\nu} \tilde{F}_{\mu\nu}^a, \quad (1)$$

with F corresponding to the field strength tensor and \tilde{F} its dual; the winding number is defined in terms of the field strength tensor as

$$Q_W = \frac{g^2}{32\pi^2} \int d^4x F_a^{\mu\nu} \tilde{F}_{\mu\nu}^a. \quad (2)$$

The term $F\tilde{F}$ is not invariant under parity transformations, \mathcal{P} , hence currents generated by quantum anomalies are referred to as \mathcal{P} -odd currents.

Study of systems with $n_5 \neq 0$, and in particular of \mathcal{P} -odd currents appeared many years ago [5, 6] with applications to early universe physics; nevertheless the possibility to observe \mathcal{P} -odd phenomena in relativistic heavy ion collisions by the celebrated Chiral Magnetic

Effect [7, 8] (see [9, 10] for reviews) has boosted the interest for the study of a medium with a net chirality, which spread from QCD to hydrodynamics and condensed matter systems [11–20], and recently has been observed in zirconium pentatelluride [21].

In order to describe systems with finite chirality in thermodynamical equilibrium, it is customary to couple the chiral chemical potential, μ_5 , to its conjugated quantity namely the chiral density quark operator, $\bar{\psi}\gamma_0\gamma_5\psi$, in the same manner one usually couples the quark number density $\bar{\psi}\gamma_0\psi$ to the conjugated quark chemical potential μ , see [22–32] and references therein. Because of quantum anomaly as well as of chirality changing processes due to finite quark condensate, μ_5 is not conjugated to a quantity which is strictly conserved; therefore usually one has to assume the system at finite μ_5 in thermodynamical equilibrium is observed on a time scale much larger than the typical time scale of the chirality changing processes.

In the context of QCD an interesting problem is the dependence of the critical temperature for chiral symmetry restoration, T_c , at $\mu_5 \neq 0$. Several calculations based on chiral models predict T_c to be lowered by μ_5 , the slope of the line $T_c(\mu_5)$ and its curvature depending on the specific model used [22–27]. On the other hand recent lattice data have shown that critical temperature increases with μ_5 [28, 29]. This behavior of $T_c(\mu_5)$ was predicted for the first time by universality arguments in [31] and it has also been found later by solving Schwinger-Dyson equations at finite μ_5 [32]. Therefore it is interesting to understand if and how chiral models can reproduce at least qualitatively the dependence of T_c versus μ_5 found on the lattice.

As it was discussed several years ago in [22], and then made even more transparent in [26], the behaviour of the critical temperature as a function of μ_5 in chiral models is related to a competition among the vacuum term and

*Electronic address: marco.ruggieri@ucas.ac.cn

†Electronic address: gxpeng@ucas.ac.cn

the thermal fluctuations. In fact although at zero temperature the chiral chemical potential acts as a catalyzer of chiral symmetry breaking, close to the critical temperature thermal fluctuations become more important at $\mu_5 \neq 0$, destroying the chiral condensate and leading to a decreasing critical temperature. The final result of the calculation was that the critical temperature decreases as μ_5 increases, even if the slope of the critical line is very small compared to that of T_c versus the baryon chemical potential.

In this competition mechanism among vacuum term and thermal excitations, the role of the divergent vacuum energy is crucial. In previous calculations the ultraviolet cutoff Λ appears explicitly in $T_c(\mu_5)$, so $T_c = T_c(\mu_5, \Lambda)$. It is therefore interesting to look for a calculation scheme in which the dependence on Λ is removed by renormalization of the vacuum energy. This is the main goal of the study presented here, in which we focus on the computation of $T_c(\mu_5)$ within a quark-meson (QM) model with renormalization prescriptions for the vacuum energy at $\mu_5 \neq 0$. We already know that renormalized vacuum energy in chiral models may affect the order of a phase transition [33, 34].

We will find that among three renormalization schemes (RSs) considered there exists one, named RS3 in the main part of the text, which predicts the increase of T_c with μ_5 , in agreement with the most recent lattice data [28, 29]. What we find interesting is that the evolution of T_c with μ_5 is achieved without the addition of any extra coupling term in the QM lagrangian, and within a simple mean field calculation. We thus find the result a nice example of how the proper treatment of the divergent vacuum energy in chiral models can affect the computation of physical quantities. The qualitative agreement of RS3 with lattice data is encouraging so we are tempted to take RS3 as the best RS among the ones studied here; however we should wait for the results of lattice simulations with masses close to the physical ones before making such a statement, and at the moment the most fair attitude is to consider the results of the present study as an exploration of the possible scenarios that the QM model can predict.

The plan of the article is as follows. In Section II we review briefly the QM model and the renormalization of the vacuum energy at $\mu_5 \neq 0$. In Section III we extend the renormalization to the case $\mu_5 \neq 0$, introducing the three RSs. In Section IV we present the main result of our study, namely the critical temperature as a function of μ_5 , obtained within the three RSs. Finally in Section V we draw our conclusions.

II. QUARK-MESON MODEL

In this Section we review very briefly the quark-meson (QM) model [35–39], focusing in particular on the thermodynamic potential in the mean field approximation and on the renormalization of the vacuum term at $\mu_5 = 0$. The QM model is defined by the following lagrangian

density

$$\mathcal{L} = \mathcal{L}_{\sigma,\pi} + \mathcal{L}_q, \quad (3)$$

where

$$\mathcal{L}_{\sigma,\pi} = \frac{1}{2} (\partial^\mu \sigma \partial_\mu \sigma + \partial^\mu \boldsymbol{\pi} \partial_\mu \boldsymbol{\pi}) - U(\sigma, \boldsymbol{\pi}), \quad (4)$$

is the lagrangian density of meson fields, with σ representing a scalar field and $\boldsymbol{\pi}$ an isotriplet pseudoscalar field. The potential $U(\sigma, \boldsymbol{\pi})$ which is responsible for spontaneous breaking of the $O(4)$ symmetry as well as for classical meson excitations spectrum is given by

$$U(\sigma, \boldsymbol{\pi}) = \frac{\lambda}{4} (\sigma^2 + \boldsymbol{\pi}^2 - v^2)^2, \quad (5)$$

which is invariant under $O(4)$ rotations in the $(\sigma, \boldsymbol{\pi})$ space. The quark content of the model is specified by the following lagrangian density:

$$\mathcal{L}_q = \bar{\psi} [i\partial^\mu \gamma_\mu - g(\sigma + i\gamma_5 \boldsymbol{\pi} \cdot \boldsymbol{\tau})] \psi + \mu_5 \bar{\psi} \gamma_0 \gamma_5 \psi, \quad (6)$$

with ψ being a quark field with Dirac, color and flavor indices. In this equation μ_5 is the chiral chemical potential, and its conjugated quantity is named the chiral charge density, $n_5 \equiv n_R - n_L$; for a system of $N_c \times N_f$ massless fermions one has the simple relation at zero temperature:

$$n_5 = \frac{N_c N_f}{3\pi^2} \mu_5^3, \quad (7)$$

which is formally equivalent to the relation among quark number density n and quark number chemical potential μ for a system of massless fermions at zero temperature, namely

$$n = \frac{N_c N_f}{3\pi^2} \mu^3; \quad (8)$$

for the case of massive fermions in Eq. (8) it is enough to replace $\mu \rightarrow \sqrt{\mu^2 - m^2}$; on the other hand between n_5 and μ_5 is more complicated and it depends on the way one treats the divergence in the vacuum energy, see the next Section.

Chiral symmetry breaking in the QM model occurs because of breaking of $O(4)$ symmetry in the meson sector down to $O(3)$ by the condensates $\langle \sigma \rangle \neq 0$ and $\langle \boldsymbol{\pi} \rangle = 0$, which is transmitted to quarks via the Yukawa coupling of the latter with the condensate background $g\langle \sigma \rangle \bar{\psi} \psi$. We assume $v = F_\pi$ in Eq. (5) which implies $\langle \sigma \rangle = F_\pi$, where F_π corresponds to the pion decay constant in the vacuum. With this choice the tree level mass spectrum of σ and π mesons is given by

$$M_\sigma^2 = 2\lambda F_\pi^2, \quad M_\pi^2 = 0. \quad (9)$$

The symmetry breaking pattern is thus equivalent to chiral symmetry breaking in QCD due to the chiral condensate, with a massive σ mode and a triplet of pions behaving as Goldstone modes.

The one-loop thermodynamic potential reads then

$$\Omega = U(\sigma, \boldsymbol{\pi}) + \Omega_q + \Omega_T, \quad (10)$$

where $U(\sigma, \boldsymbol{\pi})$ is the chiral invariant meson potential given by Eq. (5), Ω_q corresponds to the vacuum energy,

$$\Omega_q = -N_c N_f \sum_{s=\pm 1} \int \frac{d\mathbf{p}}{(2\pi)^3} \omega_s, \quad (11)$$

with

$$\omega_s = \sqrt{(p + s\mu_5)^2 + m_q^2}, \quad m_q = g\sigma; \quad (12)$$

finally Ω_T is the thermal contribution to the quark thermodynamic potential,

$$\Omega_T = -2N_c N_f \sum_{s=\pm 1} \int \frac{d\mathbf{p}}{(2\pi)^3} \log(1 + e^{-\beta\omega_s}). \quad (13)$$

The zero temperature thermodynamic potential Eq. (11) is divergent in the ultraviolet; often this divergence is treated in an effective way by introducing a certain regularization scheme, let us say an ultraviolet cutoff, and treating the cutoff as an additional parameter of the model, fixed for example by requiring that the masses of pion and sigma mesons computed within the model, as well as the chiral condensate, are in agreement with the measured values of these quantities. This choice is perfectly reasonable and legitimate, since QM model (and chiral models more generally) should be considered, at most, as low energy models for chiral symmetry breaking of QCD; it is thus natural that a cutoff appears in the calculations and treat this cutoff as a parameter. However the QM model is renormalizable, thus it is also legitimate to perform a renormalization procedure to manage the ultraviolet divergence of the vacuum energy getting rid of the cutoff introducing and physical renormalization point.

In order to prepare Ω_q in Eq. (11) for renormalization we formally expand Eq. (11) in powers of μ_5 ; we obtain

$$\Omega_q = \Omega_0 + \Omega_5, \quad (14)$$

with

$$\Omega_0 = -\frac{N_c N_f}{2\pi^2} 2 \int p^2 dp (p^2 + m_q^2)^{\frac{1}{2}}, \quad (15)$$

$$\begin{aligned} \Omega_5 = & -m_q^2 \mu_5^2 \frac{N_c N_f}{2\pi^2} \int p^2 dp (p^2 + m_q^2)^{-\frac{3}{2}} \\ & -\mu_5^4 \frac{N_c N_f}{12\pi^2}, \end{aligned} \quad (16)$$

and terms with higher powers of μ_5 vanish when momentum integration is extended to infinity. To regularize the divergent contributions we follow the strategy of [41] and introduce the functions

$$\Omega_0(s) = -\frac{N_c N_f}{2\pi^2} 2\mu^{2s} \int p^2 dp (p^2 + m_q^2)^{\frac{1}{2}-s}, \quad (17)$$

$$\begin{aligned} \Omega_5(\xi) = & -m_q^2 \mu_5^2 \mu^{2\xi} \frac{N_c N_f}{2\pi^2} \int p^2 dp (p^2 + m_q^2)^{-\frac{3}{2}-\xi} \\ & -\mu_5^4 \frac{N_c N_f}{12\pi^2}, \end{aligned} \quad (18)$$

where s and ξ are complex numbers; the regularized values of Ω_0 and Ω_5 will be obtained by analytical continuation of $\Omega_0(s)$ and $\Omega_5(\xi)$ for $s \rightarrow 0$, $\xi \rightarrow 0$ respectively. In the above equations we have introduced the mass scale μ which serves to leave the mass dimension of the integrals unchanged at $s \neq 0$ and $\xi \neq 0$; μ will appear only in the arguments of logarithms and acts as an renormalization point only in the intermediate steps of the calculation (renormalization conditions will help to remove any μ -dependence). Performing momentum integrations and making analytic continuation to $s = 0$, $\xi = 0$ we find the regularized expressions for Ω_0 and Ω_5 , namely

$$\Omega_0 = -\frac{N_c N_f}{2\pi^2} \mathcal{I}_1, \quad (19)$$

$$\Omega_5 = -m_q^2 \mu_5^2 \frac{N_c N_f}{2\pi^2} \mathcal{I}_2 - \mu_5^4 \frac{N_c N_f}{12\pi^2}, \quad (20)$$

with

$$\mathcal{I}_1 = -\frac{m_q^4}{8s} + \frac{m_q^4}{16} \left(-3 + 2\gamma_E + 2\psi(-1/2) + 4 \log \frac{m_q}{\mu} \right), \quad (21)$$

and

$$\mathcal{I}_2 = \frac{1}{2\xi} - \left(\frac{\gamma_E}{2} + \frac{\psi(3/2)}{2} + \log \frac{m_q}{\mu} \right); \quad (22)$$

in the above equations γ_E corresponds to the Euler-Mascheroni constant and $\psi(x)$ is the di-gamma function. The ultraviolet divergences of the vacuum energy appear as poles in Eqs. (19) and (20) analogously to what happens within dimensional regularization scheme.

Equation (20) shows that μ_5 adds a further divergence in the vacuum energy, which needs to be renormalized by a proper renormalization condition which has to be considered as a part of the assumptions of the model.

We now briefly sketch the standard renormalization procedure of the quantum effective potential for the QM model at zero temperature, in the case $\mu_5 = 0$. We emphasize here just the few steps which are important for the renormalization at $\mu_5 \neq 0$ which will be discussed in the following Section III. We add to Ω_q two counterterms,

$$\delta\Omega = \frac{\delta v}{2} \sigma^2 + \frac{\delta\lambda}{4} \sigma^4 \quad (23)$$

and impose the renormalization conditions [34, 40, 41]

$$\left. \frac{\partial(\Omega_q + \delta\Omega)}{\partial\sigma} \right|_{\sigma=F_\pi} = \left. \frac{\partial^2(\Omega_q + \delta\Omega)}{\partial\sigma^2} \right|_{\sigma=F_\pi} = 0, \quad (24)$$

which imply the vacuum expectation value of the σ field is not affected by the one-loop corrections and is determined by the classical potential U only. Taking into account Eqs. (23) and (24) the renormalized potential is thus given by

$$\Omega_q^{\text{ren}} = \frac{N_c N_f}{8\pi^2} \left(\frac{3}{4} m_q^4 - g^2 F_\pi^2 m_q^2 + m_q^4 \log \frac{gF_\pi}{m_q} \right). \quad (25)$$

These brief remarks about renormalization at $T = 0$ will be useful in the next Section where we describe the renormalization procedure at $\mu_5 \neq 0$.

III. RENORMALIZATION AT $\mu_5 \neq 0$

Here we discuss the renormalization procedure we use for the thermodynamic potential at zero temperature for the case $\mu_5 \neq 0$. The renormalization condition at $T = 0$ has to be considered as a part of the assumptions in the model. The divergence in Eq. (20) is transmitted to physical quantities; for example assuming that Ω_0 has been already renormalized, for $\mu_5 \ll F_\pi$ the solution for the σ condensate can be found analytically as $\sigma = F_\pi + \delta\sigma$ and expanding in powers of $\delta\sigma$. After a straightforward calculation and taking into account Eq. (24) we obtain, to the lowest order in μ_5 ,

$$\sigma = F_\pi + \mu_5^2 \frac{N_c N_f}{2\pi^2} \frac{g^2 F_\pi}{M_\sigma^2} \left(\frac{1}{\xi} + \text{finite terms} \right), \quad (26)$$

which shows that renormalization of the vacuum part with conditions in Eq. (24) still leaves a divergent contribution to the gap equation at $\mu_5 \neq 0$. Moreover by taking the second derivative of Eq. (20) with respect to the σ field, which is equivalent to compute the polarization tensor of the σ meson at zero momentum, it is possible to show that the screening mass of this meson gets a divergent contribution from the quark loop, namely

$$g^2 \Pi_s(0) = \mu_5^2 \frac{g^2 N_c N_f}{2\pi^2} \left(\frac{1}{\xi} + \text{finite terms} \right), \quad (27)$$

where $\Pi_s(Q^2)$ denotes the polarization tensor of the σ meson at zero external momentum.

The use of renormalization is not only a formal caprice: the cutoff in momentum integral removes fermion modes from the vacuum which instead might play some role at finite μ_5 . We consider for example the zero temperature chiral condensate, which can be computed easily by taking the trace of the propagator $\langle \bar{\psi}\psi \rangle = -i\text{Tr}[S(x, x)]$, with the result

$$\langle \bar{\psi}\psi \rangle = -A \int_0^\Lambda p^2 dp \left(\frac{1}{\omega_+} + \frac{1}{\omega_-} \right), \quad (28)$$

where ω_\pm are defined in Eq. (12) and $A > 0$ is a numerical constant which is not important for the discussion. In the above equation we have left an explicit ultraviolet cutoff Λ in the momentum integral. For $\mu_5 = 0$ the momentum region in which the term in parenthesis takes its largest contribution is $p \ll m_q$; on the other hand μ_5 shifts gradually this domain to higher values of $p \approx \mu_5$, as also noticed in [30], meaning that to manage properly the chiral condensate one has to include higher momenta and this is feasible either introducing an explicit dependence of Λ on μ_5 , or performing the renormalization of the ultraviolet divergence.

We will consider here three different renormalization schemes: the first one, which we name renormalization scheme 1 (RS1 in the following) where we assume the fermion vacuum term does not shift the vacuum expectation value of the σ field at $\mu_5 \neq 0$, in analogy to what we assume in the $\mu_5 = 0$ case; also the squared mass of the σ meson is assumed to be the one at $\mu_5 = 0$. The RS1 is certainly the cheapest scheme. On the other hand we will consider also less cheap renormalization schemes, named RS2 and RS3, in which we add a further counterterm at $\mu_5 \neq 0$ and impose one renormalization condition more with respect to RS1. We will show similarities as well as differences in the physical content of the several renormalization schemes, then focusing on the effect of the choice of a specific one on the restoration of chiral symmetry at finite temperature.

A. Renormalization Scheme 1

We first discuss RS1. As in the case $\mu_5 = 0$ we add to Ω_q two counterterms as in Eq. (23) and impose the renormalization conditions in Eq. (24) which imply, among other things, that $\langle \sigma \rangle$ does not take contributions from $\mu_5 \neq 0$ at $T = 0$. The renormalized Ω_5 thus reads

$$\begin{aligned} \Omega_5^{\text{ren}} = & \mu_5^2 \frac{g^2 N_c N_f}{2\pi^2} \sigma^2 \log \frac{\sigma}{F_\pi} - \mu_5^2 \frac{g^2 N_c N_f}{8\pi^2 F_\pi^2} \sigma^4 \\ & - \frac{N_c N_f}{12\pi^2} \mu_5^4. \end{aligned} \quad (29)$$

The renormalized potential in the Eq. (29) shows an explicit and finite dependence on μ_5 . It is worth noticing that the chiral density $n_5 = -\partial\Omega/\partial\mu_5$ is also finite and non-vanishing: at the global minimum of the thermodynamic potential $\sigma = F_\pi$, which is always satisfied at $T = 0$ within RS1, we obtain indeed

$$n_5 = \frac{N_c N_f}{\pi^2} \left(\frac{\mu_5^3}{3} + \frac{g^2 F_\pi^2}{4} \mu_5 \right). \quad (30)$$

For completeness we also write down the expression for the susceptibility of the chiral density $\chi_5 \equiv -d^2\Omega/d\mu^5$:

$$\chi_5 = \frac{N_c N_f}{\pi^2} \left(\mu_5^2 + \frac{g^2 F_\pi^2}{4} \right). \quad (31)$$

Equation (30) shows the increase of n_5 with increasing μ_5 as expected: adding μ_5 injects chirality in the system; nevertheless within RS1 there is no effect of such chiral density on the breaking of the $O(4)$ spontaneous symmetry breaking because of conditions (24). Therefore within RS1 the only effect of injecting chirality at $T = 0$ is a finite and condensate dependent shift of the vacuum energy.

B. Renormalization Scheme 2

The previous renormalization scheme might be satisfactory from a formal point of view; it seems however

somehow hard to accept physically because we would expect the quark loop at finite μ_5 in the grand potential to give a contribution to the σ condensate and/or to the screening mass of the σ meson, in the same way it gives a contribution at finite temperature. We therefore are tempted to check what is the effect of tilting the renormalization conditions in order to have a dependence of the σ -condensate and/or σ meson screening mass at finite μ_5 and $T = 0$.

To this end we introduce renormalization scheme 2 (RS2) which differs from RS1 because we introduce independent counterterms for the $\mu_5 = 0$ and $\mu_5 \neq 0$ parts of the vacuum energy. The idea is as follows: the chiral chemical potential introduces in the vacuum energy only one log-type divergence proportional to $\mu_5^2 \sigma^2$, and this suggests the need of one single counterterm to renormalize the divergence. Therefore to the total vacuum energy we add the counterterms

$$\delta\Omega = \frac{\delta v}{2}\sigma^2 + \frac{\delta\lambda}{4}\sigma^4 + \frac{\gamma}{2}\mu_5^2\sigma^2 \quad (32)$$

$$\equiv \Omega_{c.t.} + \frac{\gamma}{2}\mu_5^2\sigma^2, \quad (33)$$

instead of Eq. (23). The explicit μ_5^2 in the above equation is just a matter of convention. We use $\Omega_{c.t.}$ in Eq. (33) to renormalize the $\mu_5 = 0$ potential, assuming the renormalization conditions

$$\left. \frac{\partial(U + \Omega_{c.t.} + \Omega_0)}{\partial\sigma} \right|_{\sigma=F_\pi} = 0, \quad (34)$$

$$\left. \frac{\partial^2(U + \Omega_{c.t.} + \Omega_0)}{\partial\sigma^2} \right|_{\sigma=F_\pi} = M_\sigma^2, \quad (35)$$

with $M_\sigma^2 = 2\lambda F_\pi^2$. Conditions (34) and (35) are formally identical to the ones used in the previous Section to renormalize the vacuum energy at $\mu_5 = 0$: they are thus enough to remove quadratic and log the divergences appearing in Ω_0 .

We now use the γ -counterterm in Eq. (33) to renormalize the divergence in Ω_5 in Eq. (20). To this end we assume the renormalization condition

$$\left. \frac{\partial(\Omega_5 + \gamma\mu_5^2\sigma^2/2)}{\partial\sigma} \right|_{\sigma=F_\pi} = 0. \quad (36)$$

The above condition is enough to remove the divergence of Ω_5 :

$$\Omega_5^{\text{ren}} = \mu_5^2 \frac{g^2 N_c N_f}{2\pi^2} \sigma^2 \left(-\frac{1}{2} + \log \frac{\sigma}{F_\pi} \right) - \frac{N_c N_f}{12\pi^2} \mu_5^4, \quad (37)$$

and together with conditions (34) and (35) it implies that $\sigma = F_\pi$ at $T = 0$: the chiral chemical potential within RS2 does not shift the expectation value of σ . However the second derivative of Ω_5^{ren} with respect to σ is finite, therefore the screening mass of the σ meson gets a finite μ_5 dependence given by

$$m_\sigma^2 = M_\sigma^2 + \frac{g^2 N_c N_f}{\pi^2} \mu_5^2. \quad (38)$$

For completeness we compute also the chiral density and its susceptibility within RS2 at zero temperature; we find

$$n_5 = \frac{N_c N_f}{\pi^2} \left(\frac{\mu_5^3}{3} + \frac{g^2 F_\pi^2}{2} \mu_5 \right), \quad (39)$$

and

$$\chi_5 = \frac{N_c N_f}{\pi^2} \left(\mu_5^2 + \frac{g^2 F_\pi^2}{2} \right). \quad (40)$$

Before going ahead we would like to comment that we have checked that instead of $\sigma = F_\pi$ in Eqs. (34), (35) and (36) we can use $\sigma = S$ with any real value of S and still the renormalization procedure works. This opens the possibility to add by hand in RS2 a dependence $\sigma(\mu_5)$ which is certainly feasible but it should rely on the choice of an ansatz: we prefer to rely not on an ansatz hence we consider in this study only the solution $\sigma = F_\pi$ within RS2.

It is also worth noticing that the chiral condensate $\langle \bar{\psi}\psi \rangle = \partial\Omega/\partial m_0$ has a μ_5 dependence within RS2 as well as RS1 at $T = 0$ even if the σ condensate does not depend on μ_5 . Since we are interested here only to a qualitative behaviour of $\langle \bar{\psi}\psi \rangle$ versus μ_5 we limit ourself to a phenomenological treatment of the divergence and introduce an ultraviolet cut off Λ in the relevant momentum integrals. Of course this naive procedure does not affect at all the results of this Section, and we use it only with the purpose to illustrate qualitatively the behaviour of the chiral condensate at finite μ_5 . A straightforward calculation shows that

$$\langle \bar{\psi}\psi \rangle = \langle \bar{\psi}\psi \rangle_0 + \mu_5^2 m_q \frac{N_c N_f}{2\pi^2} \left(3 - 2 \log \frac{2\Lambda}{m_q} \right). \quad (41)$$

where $\langle \bar{\psi}\psi \rangle_0$ is the chiral condensate at $\mu_5 = 0$, m_q is the solution of the gap equation and Λ is an ultraviolet cutoff, $\Lambda \gg m_q$. We notice that the μ_5 -dependent term in the above equation shows an explicit log-type divergence; the same kind of divergence appears in full QCD using the lattice regularization [28, 29]. Equation (41) shows that the chiral chemical potential increases the magnitude of the chiral condensate, its correction being negative for $\Lambda \gg m_q$. This is a clear evidence that μ_5 favours chiral symmetry breaking at $T = 0$. The catalysis at small temperature with $\mu_5 \neq 0$ has been found for the first time in [23] using an effective model with a coupling to the Polyakov loop [42]; then it has also been found in several other model calculations [22, 25, 26]; recently this catalysis at zero temperature has been described very nicely in terms of BCS-like instability of the Fermi surface at $\mu_5 \neq 0$ [30]. Chiral condensate has been measured recently on the lattice at finite μ_5 and finite temperature, and it has been found that also at temperatures below the critical temperature $\langle \bar{\psi}\psi \rangle$ increases with μ_5 for the case of staggered fermions [29], thus supporting the catalysis of chiral symmetry breaking at finite μ_5 in qualitative agreement with Eq. (41).

C. Renormalization Scheme 3

Next we turn to renormalization scheme 3 (RS3) which slightly differs from RS2 because it allows a dependence of the σ condensate on μ_5 at $T = 0$. In fact within RS2 the effect of μ_5 at zero temperature is a shift of the screening mass of the σ meson, at the same time assuming $\sigma = F_\pi$ for any value of μ_5 . Once again this choice is certainly satisfactory from a formal point of view, but probably still lacks some physical content because if μ_5 acts as a catalyzer of spontaneous chiral symmetry breaking, then we would expect the σ condensate to increase with μ_5 .

Within RS3 we take counterterms as in RS2, see Eqs. (32) and (33) and assume the conditions (34) and (35). On the other hand, instead of condition (36) which eventually constraints $\sigma = F_\pi$ for any μ_5 , we assume the less severe condition

$$\left. \frac{\partial^2(\Omega_5 + \gamma\mu_5^2\sigma^2/2)}{\partial\sigma^2} \right|_{\sigma=F_\pi} = 0. \quad (42)$$

The above equation is equivalent to a minimal subtraction scheme for the polarization tensor of the σ meson, in which the dressed propagator for this quantum fluctuation at $T = 0$ is given by

$$D_\sigma^{-1}(Q^2) = Q^2 - M_\sigma^2 + g^2 [\Pi_s(\sigma, Q^2) - \Pi_s(F_\pi, 0)], \quad (43)$$

where Π_s corresponds to the polarization tensor; within RS3 whenever in the ground state $\sigma = F_\pi$ then the screening mass of the σ meson does not take contribution from the zero temperature part of the quark loop.

The above conditions are enough to remove the divergence of Ω_5 which in RS3 reads

$$\Omega_5^{\text{ren}} = \mu_5^2 \frac{g^2 N_c N_f}{2\pi^2} \sigma^2 \left(-\frac{3}{2} + \log \frac{\sigma}{F_\pi} \right) - \frac{N_c N_f}{12\pi^2} \mu_5^4. \quad (44)$$

The present renormalization scheme is quite interesting because it permits the computation of the μ_5 dependence of the σ condensate at $T = 0$, as well as the shift of the screening mass of the σ meson. Although the previous equations are valid for any value of μ_5/σ we limit ourselves to a solution of the gap equation for the σ condensate for small values of μ_5/F_π because in this case we obtain simple analytical relations, and more important the role of the vacuum term on the critical temperature will be more transparent.

The σ condensate in the limit $\mu_5/F_\pi \ll 1$ is computed by minimizing $\Omega = U + \Omega_0^{\text{ren}} + \Omega_5^{\text{ren}}$ with respect to σ , with U and Ω_0^{ren} defined in Eqs. (5) and (25), assuming the ansatz $\sigma = F_\pi(1 + \delta)$ with $\delta \ll 1$ and expanding to the lowest non trivial order in δ . The calculation is straightforward and not informative so we limit ourself to quote the final result, namely

$$\sigma = F_\pi \left(1 + \frac{\mu_5^2}{F_\pi^2} \frac{g^2 N_c N_f}{\pi^2} \frac{F_\pi^2}{M_\sigma^2} \right), \quad \text{RS3.} \quad (45)$$

	RS1	RS2	RS3
$\langle\sigma\rangle$	F_π	F_π	$F_\pi (1 + a\mu_5^2/F_\pi^2)$
m_σ^2	M_σ^2	$M_\sigma^2 + b\mu_5^2$	$M_\sigma^2 + c\mu_5^2$

Table I: $\langle\sigma\rangle$ and screening mass of the σ -meson at zero temperature within the three renormalization schemes discussed in the text. The value of the positive constants a , b and c is not important for the purpose of the Table and their exact values can be found in the main text. F_π and M_σ denote the pion decay constant and the σ -meson screening mass in the vacuum.

The above equation shows the σ condensate increases with increasing μ_5 as expected. We can also compute the shift of the screening mass of the σ meson by computing the second derivative of Ω with respect to σ at the global minimum of the potential (45). We find

$$m_\sigma^2 = M_\sigma^2 + \frac{\mu_5^2}{M_\sigma^2} \frac{g^2 N_c N_f}{\pi^2} \left(3M_\sigma^2 - \frac{g^4 N_c N_f F_\pi^2}{\pi^2} \right). \quad (46)$$

For completeness we report the zero temperature relation among chiral density and chemical potential within RS3 which is easily obtained from Eq. (44), namely

$$n_5 = \frac{N_c N_f}{\pi^2} \left(\frac{\mu_5^3}{3} + \frac{3g^2 F_\pi^2}{2} \mu_5 \right); \quad (47)$$

we also compute the zero temperature chiral density susceptibility, whose calculation is elementary but requires some care because the condensate within RS3 depends on μ_5 and this dependence has to be taken into account when the second derivative of the vacuum energy with respect to μ_5 is performed:

$$\chi_5 = \frac{N_c N_f}{\pi^2} \left[\mu_5^2 \left(1 + \frac{6g^4 F_\pi^2}{M_\sigma^2} \frac{N_c N_f}{\pi^2} \right) + \frac{3g^2 F_\pi^2}{2} \right]. \quad (48)$$

For sake of reference in Table I we summarize the physical content of the three renormalization schemes discussed above. We have not specified the values of the positive proportionality constants because the only purpose of the Table is to collect the relevant similarities and differences of the three renormalization schemes. Before going ahead we notice that n_5 and χ_5 depend on the renormalization scheme used: this is not surprising because, even if they correspond to physical observables, before renormalization they are divergent and the three RSs, which formally correspond to different subtractions in the bare expressions for n_5 and χ_5 , lead to three different models as we have summarized in Table I.

IV. THE CRITICAL TEMPERATURE

We now focus on the main goal of the present study, namely to understand the effect of a renormalized vacuum term on the critical temperature for chiral symmetry

restoration at finite μ_5 . Thanks to the analytical results discussed in the previous Section, and to the expansion of Ω_T in Eq. (13) for large temperature, we can observe in a transparent way the effect we look for.

The effect of the renormalization scheme on the critical temperature can be understood by the computation of the second order Ginzburg-Landau coefficient. For temperatures close to the critical temperature we write $\Omega = \beta_2 \sigma^2$; the coefficient β_2 is negative in the chirally broken phase and vanishes at the critical temperature. Extracting the $O(\sigma^2)$ term from the thermodynamic potential we find

$$\beta_2 = \beta_{2,0} + \beta_2(T), \quad (49)$$

where

$$\begin{aligned} \beta_{2,0} = & -\frac{\lambda F_\pi^2}{2} - \frac{g^4 N_c N_f}{8\pi^2} F_\pi^2 \\ & + g^2 \mu_5^2 \frac{N_c N_f}{2\pi^2} \log \frac{\sigma}{F_\pi} - \xi g^2 \mu_5^2 \frac{N_c N_f}{2\pi^2} \end{aligned} \quad (50)$$

and

$$\begin{aligned} \beta_2(T) = & g^2 T^2 \frac{N_c N_f}{12} \\ & + g^2 \mu_5^2 \frac{N_c N_f}{2\pi^2} \left(\log \frac{\pi T}{\sigma} - \gamma_E - \frac{1}{2} \right), \end{aligned} \quad (51)$$

where $\beta_{2,0}$ and $\beta_2(T)$ correspond to the vacuum and thermal fluctuations contributions to β_2 respectively; for $\beta_2(T)$ we have used the high temperature expansion quoted in [26] paying attention to the different definition of the constituent quark mass; finally ξ in Eq. (50) depends on the RS we use:

$$\xi = 0, \quad \text{RS1}, \quad (52)$$

$$\xi = \frac{1}{2}, \quad \text{RS2}, \quad (53)$$

$$\xi = \frac{3}{2}, \quad \text{RS3}. \quad (54)$$

The log term in the above equations exactly cancels out leaving a regular expression in the $\sigma \rightarrow 0$ limit.

Before commenting the effect of $\xi \neq 0$ it is useful to remind that an inspection of Eqs. (50) and (51) shows the competition of the vacuum and the thermal fluctuations in order to determine the effect of μ_5 on the critical temperature at $\xi = 0$. As a matter of fact for $\sigma \rightarrow 0$ the vacuum term gives a negative contribution to β_2 while the thermal part gives a positive contribution, hence favouring restoration of chiral symmetry. When they sum up the thermal contribution wins the competition and the net effect of β_2 is to lower the critical temperature. For $\xi = 0$ in RS1 the $O(\mu_5^2)$ the vacuum term does not add any further contribution to β_2 leading eventually to a decreasing critical temperature μ_5 with increasing μ_5 .

On the other hand when $\xi > 0$ there is an additional negative contribution to β_2 due to the vacuum term,

which in fact can change the overall sign of the $O(\mu_5^2)$ leading to an increase of the critical temperature at finite μ_5 . We can show this by an expansion of β_2 around the solution $T_c = T_c^0 + \delta T$ in powers of δT , with T_c^0 being the critical temperature at $\mu_5 = 0$, and retaining only the lowest order in δT and μ_5^2 . Taking into account that $\beta_2(T_c^0) = 0$ at $\mu_5 = 0$ we find

$$\beta_2 \approx \delta T \frac{g^2 N_c N_f}{6} T_c^0 + \mu_5^2 \frac{g^2 N_c N_f}{2\pi^2} \left(\log \frac{\pi T_c^0}{F_\pi} - \tilde{\xi} \right) \quad (55)$$

with $\tilde{\xi} = \xi + \gamma_E + 1/2$. The above equation shows that the thermal part entering through the log-term, and the vacuum part can compete at $\xi \neq 0$; the overall sign of the $O(\mu_5^2)$ correction to β_2 depends on the numerical values of T_c^0 and ξ , and for ξ large enough the correction to β_2 is negative, hence μ_5 leads to an increase of the critical temperature. By solving for δT the equation $\beta_2 = 0$ we find the critical temperature as a function of μ_5 , namely

$$T_c(\mu_5) = T_c^0 + \mu_5^2 \frac{3}{\pi^2 T_c^0} \left(\xi + \gamma_E + \frac{1}{2} - \log \frac{\pi T_c^0}{F_\pi} \right), \quad (56)$$

which together with Eq. (55) represents the main result of the present article. In the above equation T_c^0 is the only free parameter once the renormalization scheme has been specified, therefore we can predict the behaviour of $T_c(\mu_5)$ by varying T_c^0 within a reasonable range.

In Fig. 1 we plot the critical temperature as a function of μ_5 for the three renormalization schemes discussed in this article: upper panel corresponds to $T_c^0 = 150$ MeV and lower panel to $T_c^0 = 180$ MeV. The chiral chemical potential leads to a lowering of T_c within RS1 and RS2, in which the condensate at $T = 0$ has no dependence on μ_5 . On the other hand within RS3 the critical temperature increases with μ_5 for both values of T_c^0 . The results obtained within RS1 and RS2 are in agreement with previous model calculations [22–24, 26] but in disagreement with recent lattice results [28, 29]. On the other hand the scenario obtained within RS3 is in qualitative agreement with the latter data.

V. CONCLUSIONS

In this Article we have studied the catalysis of chiral symmetry breaking due to a chiral chemical potential, μ_5 , within a renormalized quark-meson (QM) model. The vacuum term at $\mu_5 \neq 0$ needs to be treated with care because μ_5 adds a log-type divergence to the vacuum energy which is transmitted to physical quantities. In order to deal with ultraviolet divergences we have introduced three different renormalization schemes (RSs) at $\mu_5 \neq 0$, whose physical content is summarized in Table I.

We have then computed the critical temperature, T_c , for chiral symmetry restoration as a function μ_5 . We have found that within RS1 and RS2 the critical temperature decreases with increasing μ_5 , in agreement with previous model calculations. On the other hand within RS3 we

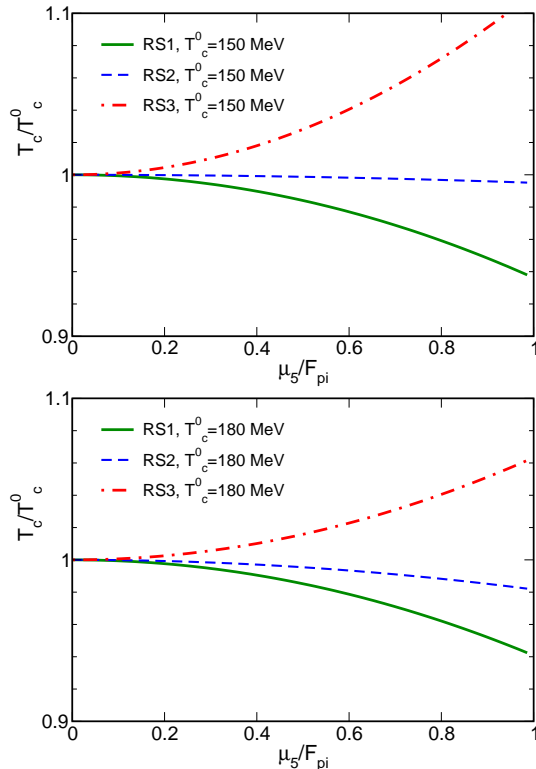


Figure 1: Critical temperature as a function of μ_5 for the three renormalization schemes discussed in this article: upper panel corresponds to $T_c^0 = 150$ MeV and lower panel to $T_c^0 = 180$ MeV.

have found that $T_c(\mu_5)$ increases with μ_5 , in agreement with recent lattice estimates of the same quantity as well as with previous analysis based on universality [31] and solution of Schwinger-Dyson equation [32].

The reason of the discrepancy among the RSs is easy to understand. Within the model at hand, as well as other simple chiral models, the behaviour of $T_c(\mu_5)$ is understood as a competition between the vacuum energy on the one hand, and on the thermal part of the thermodynamic potential on the other hand [22, 26]. Expanding the thermodynamic potential in power series of σ in the vicinity of the phase transition, namely $\Omega = \beta_2 \sigma^2$ with β_2 given by Equations (49), (50) and (51), it is clear that the vacuum term at $\mu_5 \neq 0$ near the phase transition tends to lower the value of β_2 , hence pushing the critical temperature towards larger values; on the other hand the contribution of the thermal fluctuations to β_2 is positive, hence these tend to lower the critical temperature. In previous model calculations it has been always found that eventually the competition is won by the thermal fluctuations because the vacuum term was not strong enough. We have obtained the same scenario within the RS1 and RS2. On the other hand within RS3 the vacuum energy gets an extra negative term from renormalization which makes it strong enough to compete against thermal fluc-

tuations and push $T_c(\mu_5)$ towards larger values. This is clearly represented by Eq. (56) in which ξ depends on the RS used: $\xi = 0, 1/2$ and $3/2$ for RS1, RS2 and RS3 respectively.

We would like to remark that even if T_c represents a physical quantity in principle measurable, in our calculations it depends explicitly on the RS because the three RSs introduced here correspond eventually to three different models, in which the ground state at $T = 0$ reacts in a different way to μ_5 .

The qualitative agreement of RS3 with lattice data is encouraging so we are tempted to take RS3 as the best RS among the ones studied here; however we should wait for the results of lattice simulations with masses close to the physical ones before making such a statement, and at the moment the most fair attitude is to consider the results of the present study as an exploration of the possible scenarios that the QM model can predict.

In conclusion, the lesson we have learned from this study is that the vacuum energy in chiral models has to be treated with care; a similar call for a proper treatment of the vacuum energy was given in [34] in which it was shown that the order of the phase transition in a magnetic field can change depending on the way the vacuum energy is renormalized. In the context of strong interactions at finite μ_5 we have found that predictions about the behaviour of the critical line at finite μ_5 depend on the way the divergence of the vacuum energy is treated. Moreover we have shown that a simple chiral model like the QM model is capable to reproduce qualitatively the recent lattice data about $T_c(\mu_5)$, within a mean field approximation, and without the addition of any exotic ingredient, once a renormalization scheme is chosen. The results presented here are thus encouraging and suggest to further investigate properties of strongly interacting matter with chiral chemical potential by the QM model.

Finally we would like to mention that a possible future direction for investigation is the study of in-medium properties of the system with finite chirality, using for example the strategies of [43–46]. It is also worth to mention that in chiral models it is often assumed a local interaction which leads to a momentum-independent fermion self-energy at tree level, in turn implying the consideration of all momentum modes in the chiral condensate at finite temperature and these might affect the role of the thermal fluctuations on the critical temperature. In our opinion it is urgent to understand the effect of a momentum-dependent quark self-energy on the evolution of the chiral condensate at finite μ_5 and T . Works along these directions are already in progress and we plan to report on these topics in the near future.

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